Improvement of a Large Analytical Model Using Test Data

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A method has been developed which uses measured normal modes and natural frequencies to improve an analytical mass and stiffness matrix model of a structure. The method directly identifies, without iteration, a set of minimum changes in the analytical matrices which force the eigensolutions to agree with the test measurements. An application is presented in which the analytical model had 508 degrees of freedom and 19 modes were measured at 101 locations on the structure. The resulting changes in the model are judged to be small compared to expectations of error in the analysis. Thus, the improved model is accepted as a reasonable model of the structure with improved dynamic response characteristics. In addition, it is shown that the procedure may be a useful tool in identifying apparent measured modes which are not true normal modes of the structure.

Nomenclature

A	= analytical matrix
C	= matrix of changes
I	= identity matrix
K,M	= full improved stiffness and mass matrices $(n \times n)$
K_A,M_A	= full analytical K,M matrices $(n \times n)$
K_1,M_1	= partitions of K_A,M_A corresponding to test coordinates
K_2,M_2	= partitions of K_A,M_A corresponding to coupling elements
K_4,M_4	= partitions of K_A,M_A corresponding to unmeasured coordinates
m	= number of measured modes
m_A	$= \Phi^T M_A \Phi, m_{A_{ii}} = 1, (m \times m)$
n	= number of degrees of freedom in the model
$\delta_1,\delta_2,\delta_3$	= measures of changes, Eqs. (15-17)
ϵ	= matrix norm, sum of the squares of all the elements
Φ	= rectangular modal matrix, normalized $(n \times m)$
ϕ_i	= ith mode, ith column of Φ
ϕ_{1_i},ϕ_{2_i}	= measured and unmeasured partitions of ϕ_i
Ω	= diagonal matrix of measured natural frequencies $(m \times m)$
ω_i	= natural frequency of ith mode $= \Omega_{ii}$
$\|\|(\ )\|\|$	= sum of the squares of all elements of matrix ( )

Introduction

THE use of dynamic test data, which is relatively crude, to improve a carefully formulated detailed analysis of a structure may appear to be inappropriate. However, if analytical predictions do not match observed behavior, some action should be taken. Vibration testing is often used to "validate" an analytical model. If the test and analysis are in subjective agreement, more credance is given to the analytical model. If they do not agree, the choices of action range from accepting the analysis or the test to modifications of either or both.

The determinations of structural natural frequencies and modes from vibration test data involves numerous sources of discrepancies or errors. Some of these are inexact equipment calibration, excessive noise, equipment malfunction, manufacturing variations, misinterpretation of data, incorrect transducer locations, operation in a region of nonlinearity, as well as the use of inappropriate modal identification algorithms.

Analytical models also contain errors. Some of their sources are inappropriate theoretical assumptions, inaccuracies in estimated material properties, insufficient or incorrect modeling detail, typographical computer input errors, and improper application of solution algorithms.

The method which is applied in this paper does not address the questions of improving testing methods or improving analysis procedures. It is assumed that data is available which is representative of the dynamic behavior of the structure. It is also assumed that the analysis is a "good" representation of the structure. When it is found that the analytical model does not predict the observed behavior, minimum changes in the model are determined which will cause it to predict the results of the test. If these changes are assessed to be "small," i.e., within the expected uncertainties in the model, then the improved model may be considered to be a better dynamic representation of the structure. This model may then be used with greater confidence for the analysis of the structure under different boundary conditions or with physical structural changes.

Prior to the acceptance of test data and an analytical model, it would be quite reasonable to use major discrepancies to intuitively identify and correct obvious test and analysis errors.

Particular features of the method presented are: the analytical model may have more degrees of freedom than the test data; the method is computationally efficient and requires no eigensolutions or iterative computations; the resulting model will predict exactly the specified modes and frequencies; it has been successfully applied to a large, realistic aerospace structure; it can be used to identify invalid "measured" modes.

Discussion of Related Research

In recent years there have been a number of publications of methods which use test data to identify or improve an analytical model of a structure. Bibliographies may be found in Refs. 1 and 2. This discussion shall be limited to certain particular methods which use measured normal modes and natural frequencies for this purpose. One of the earliest publications in this category is by Rodden[3] who used such data to directly identify structural influence coefficients.

Several publications are worthy of note and are representative of methods which use the same data as this paper (i.e., an incomplete set of measured modes and frequencies) but different mathematical approaches. Collins et al.[4] presents a statistically based iteration procedure. Chen and Garba[5] and Grossman[6] use perturbation methods to

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improve the efficiency of iteration. These methods have characteristics that limit their application to models with a relatively small number of degrees of freedom. Probably the most serious limitation is the requirement that a one-to-one relationship must be established between the modes of the model and the measured modes. For a model of significant size this correspondence will not be obvious. An improper selection will cause a particular analytical mode to be modified to match the wrong measured mode. This can result in large and unrealistic changes in parameters and poor convergence. Grossman[6] addresses this problem by obtaining correlation coefficients for all combinations of analytical and measured modes. For large models, there are usually many more analytical modes than measured and this procedure also may not be satisfactory.

These methods require repeated solutions of the eigenvalue equation or repeated estimations based on a perturbation method. Especially in the case of Ref. 4, this procedure will be quite costly for large problems. In Refs. 5 and 6 a set of physical parameters are selected for variation. For large problems, the selection of a limited set of appropriate parameters is beyond the intuitive capabilities of an analyst. Omitting sensitive parameters can result in large and unrealistic changes in those being varied. In any case, there is no reason for concluding that the resulting parameters have any physical significance.

The works of Collins et al.,[4] Chen and Garba,[5] and Grossman[6] are innovative and could be useful for relatively small models. The problems discussed for application to large models could probably be minimized through further development. The method described in this paper, in contrast, avoids these difficulties from the start.

Research Leading to Present Method

The conceptual basis of the approach presented here was published in Ref. 7 with a practical application in Ref. 8. This method improves an analytical mass matrix by finding the smallest changes which make a set of measured modes orthogonal using a pseudoinverse algorithm. In addition, an incomplete stiffness matrix is formed by summing the contributions of the measured modes and with the use of the improved mass matrix. This method does not iterate or use eigensolutions and the resulting model is consistent with the test data. The stiffness matrix is useful but, because the dominant higher modes are unknown, it does not resemble a true stiffness matrix of the structure in the magnitude of the elements. This procedure is also not suitable for large models, since it involves operations using matrices of the order of the number of unknowns, i.e., approximately one half the square of the order of the analytical model.

Baruch and Bar Itzack[9,10] formulated a procedure using Lagrange multipliers for minimizing changes in matrices to satisfy specified constraints. This approach to correcting an analytical model is to assume the mass matrix is correct and to modify the measured modes to make them orthogonal. Then, the stiffness matrix is corrected to satisfy the eigenequation using the analytical mass, the measured frequencies, and the corrected modes. The assumption that the mass is correct is questionable, especially for a dynamic model which is often an approximate reduced version of a much larger model. The arbitrary modification of the measured modes also is unlikely to have any physical meaning. These questions have been discussed in Ref. 11. The mathematical approach of Baruch, however, was adapted to the modification of the mass matrix using the modal displacements as determined in tests.[12] An alternate derivation of the stiffness correction algorithm was carried out by Wei.[13] Baruch has also presented a general discussion of the formulation of related procedures using various "reference bases."[14]

The method of this paper performs the analysis of Ref. 12 to improve the mass matrix and then applies the concept of Refs. 10, 11, and 13 to improve the stiffness matrix. There is a limitation in the stated procedures in that the measured modal displacements are known only at the points of measurement and, thus, it is only possible to improve a model that has been reduced to these coordinates. In order to allow for the correction of a larger model, it is necessary to obtain an estimate of the modal displacements that are not measured. This could have been performed by a geometrical interpolation, however, for a large three-dimensional structure such a procedure would be inadequate. Instead a "physical interpolation" is carried out using the formulation of Ref. 15 which relates subsets of modal displacement through the eigenvalue equation.

The resulting method is carried out in three steps: the "full" modes are estimated; the mass matrix is corrected; and the stiffness matrix is corrected. The improved model is exactly consistent with the measured modes and frequencies and does not require iteration or eigenanalysis. This procedure was applied previously to simulated test data and published in Ref. 16. The theoretical development and application to a large model is described in the following.

Description of the Problem

This procedure is applicable where an analytical model of an existing structure is available in the form of mass and stiffness matrices. In addition, ground vibration tests have been performed and a set of normal modes and natural frequencies have been identified. It is assumed that the structure behaves in a linear fashion and is damped proportionally so that the measured modes are real. A decision has been made that the measured modes and frequencies are representative of the dynamics of the structure and it is desired to modify the analytical model so that its predictions will include the measured modes and frequencies. It is further assumed that the analytical model is a "good" representation of the structure and that only "small" changes will be required.

Two basic relationships are to be satisfied: modal orthogonality and the eigenvalue equation.

$$\Phi^T M \Phi = I \tag{1}$$

$$K\Phi = M\Phi\Omega^2 \tag{2}$$

In these equations M,K are unknown and Φ,Ω^2 represent the test data. Φ is an $n \times m$ matrix; Ω^2 an $m \times m$ diagonal matrix; and M,K square, symmetric matrices of order n. M_A,K_A are good approximations of M,K and generally represent the geometrical and material properties of the structure. The strategy for obtaining an improved model is to find M,K that satisfy Eqs. (1) and (2) and deviate as little as possible from M_A,K_A by some reasonable criteria.

When the results are obtained, it is left to the analyst to determine if the improved matrices are reasonable representations of the structure. Ideally, a solution should be acceptable only if the changes fall within the ranges of expected errors in the analysis; are physically reasonable (e.g., no negative masses); and do not significantly vary the known rigid body characteristics (e.g., total mass, center of gravity, moments of inertia). If the solution is judged to be unacceptable, and since these changes are minimum, it must be concluded that either the model or the test or both contain significiant errors and that a re-evaluation is necessary.

Theoretical Analysis of Analytical Model Improvement

The theory of analytical model improvement (AMI) is described fully in Ref. 17. A summary is presented here for convenience.

Full Mode Computation

If the modal displacements are measured at a subset of the coordinates of the analytical model, the first step in this process is to obtain an estimate of the modal displacements at all the coordinates for each measured mode.

The M_A and K_A matrices are to be ordered so that the upper coordinates correspond to those at which the measurements were made. Then Eq. (2) may be written for each mode in partitioned form[15] as follows:

$$\left\{\begin{bmatrix} K_1 & K_2 \\ K_2^T & K_4 \end{bmatrix} - \omega_i^2 \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_4 \end{bmatrix}\right\}\begin{Bmatrix} \phi_{1_i} \\ \phi_{2_i} \end{Bmatrix} = 0 \qquad (3)$$

From Eq. (3) one may obtain

$$\phi_{2_i} = -(K_4 - \omega_i^2 M_4)^{-1}(K_2^T - \omega_i^2 M_2^T)\phi_{1_i} \qquad (4)$$

If M_A, K_A are reasonable representations of the structure, Eq. (4) may be considered to be a physically meaningful interpolation relationship between the unmeasured elements of the mode, ϕ_{2_i}, and those elements which were measured, ϕ_{1_i}.

Equation (4) involves the inversion of a large matrix for each mode and may be a limiting factor on the application of this procedure. This step is by far the most costly in the AMI analysis. In the implementation of the computer program (discussed in Refs. 16 and 17), an option to ignore the frequency terms is available. This results in the well-known Guyan reduction relationship[18] which is known to be exact only at $\omega = 0$ (see Ref. 19, for example). An intermediate approximation could use a series solution when ω is small, as in Kidder.[15] Both of these alternatives are more efficient than Eq. (4) since only one inversion is required for all the modes.

It is noted that regardless of the approximation used ϕ_{1_i} (which is measured) is not changed and at the completion of the process the improved analytical model will predict the measured values. After the full modes are calculated, Φ is formed and all the remaining analyses operate with the full matrices.

Mass Matrix Improvement

The next step in the procedure is to obtain an improved mass matrix. In Eq. (1), Φ is now known and M is unknown. A unique solution is to be obtained by selecting a measure of differences between M and M_A and minimizing this quantity. It is inappropriate to simply minimize the absolute differences between the elements, since there may be large variations in the order of magnitude of the elements of M_A. If M and M_A were diagonal, an appropriate function would be the sum of the squares of the ratios of the changes to the original elements. A more general expression is used which reduces to the preceding for diagonal matrices and is as follows:

$$\epsilon = \| M_A^{-1/2}(M - M_A)M_A^{-1/2} \| \qquad (5)$$

Treating Eq. (1) as a constraint and using the method of Lagrange multipliers, a function Ψ may be written

$$\Psi = \epsilon + \sum_{i=1}^{m}\sum_{j=1}^{m} \lambda_{ij}(\Phi^T M \Phi - I)_{ij} \qquad (6)$$

Equation (6) is differentiated with respect to the elements of M and set to 0. Then using Eq. (1) to evaluate the undetermined multipliers, λ_{ij} yields a matrix M which minimizes ϵ and satisfies Eq. (1), the orthogonality condition. This procedure results in[12,17]

$$M = M_A + M_A \Phi m_A^{-1}(I - m_A)m_A^{-1}\Phi^T M_A \qquad (7)$$

where $m_A = \Phi^T M_A \Phi$, the normalized generalized mass matrix obtained from the analytical mass matrix and the full modes obtained from Eq. (4). (An application of this form of m_A to obtain an improved orthogonality check of test modes is discussed in Ref. 20.)

Equation (7) requires a modest computational effort since the only inversion is of m_A, which is of order m. The M matrix is inherently symmetrical and, thus, such an additional constraint is not required. Note, also, that $M_A^{-1/2}$ which appears in Eq. (5) need not be computed.

Stiffness Matrix Improvement

The stiffness matrix improvement essentially is equivalent to that of Baruch.[9] In Eq. (2), M, Φ, and Ω^2 are now known and K is the only unknown. In a manner similar to the mass matrix improvement, it is possible to derive an improved stiffness matrix. The constraint equations may be summarized as follows:

$$K\Phi - M\Phi\Omega^2 = 0 \qquad (8)$$

$$\Phi^T K \Phi - \Omega^2 = 0 \qquad (9)$$

$$K - K^T = 0 \qquad (10)$$

where Eq. (8) is the eigenvalue equation [Eq. (2)]. Equation (9) is a result of premultiplying Eq. (8) by Φ^T but its inclusion simplifies the derivation, and Eq. (10) is necessary since Eq. (8) is not symmetric.

The function to be minimized may be justified as in Eq. (5) and is

$$\epsilon = \| M^{-1/2}(K - K_A)M^{-1/2} \| \qquad (11)$$

Defining an undetermined multiplier matrix for each of the constraint equations $(\Lambda_K, \Lambda_O, \Lambda_s)$, the Lagrangian function may be formed as follows:

$$\Psi = \epsilon + \sum_{i=1}^{n}\sum_{j=1}^{m} \Lambda_{K_{ij}}(K\Phi - M\Phi\Omega^2)_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{m} \Lambda_{O_{ij}}(\Phi^T K \Phi - \Omega^2)_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} \Lambda_{s_{ij}}(K - K^T)_{ij} \qquad (12)$$

Equation (12) is differentiated with respect to the elements of K and set to 0. Using the constraint equations to eliminate the undetermined multipliers yields[9,13,17]

$$K = K_A + (\Delta + \Delta^T) \qquad (13)$$

where

$$\Delta = \tfrac{1}{2} M\Phi(\Phi^T K_A \Phi + \Omega^2)\Phi^T M - K_A \Phi\Phi^T M \qquad (14)$$

Equation (14) is evaluated readily using simple matrix operations and no inversions.

Summary of AMI Procedure

The application of the method is straightforward and is performed in three steps.

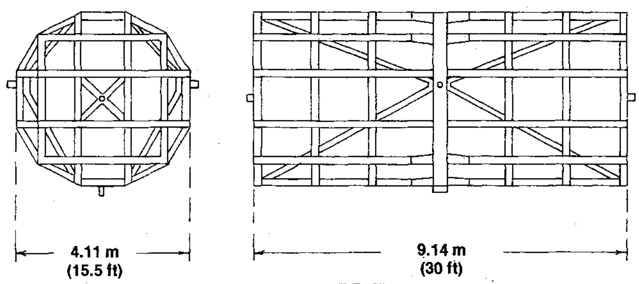

Fig. 1 Schematic of LDEF structure.

1) Equation (4), which uses analytical M_A,K_A and the measured modal displacements ϕ_{I_i} and natural frequencies ω_i, is used to obtain the "full" modal vectors ϕ_i, from which Φ is formed.

2) M_A and the now known Φ are used in Eq. (7) to obtain M which satisfies the orthogonality relationship between the modes.

3) K_A and the known M,Φ,Ω^2 are used in Eqs. (13) and (14) to obtain K which is symmetric and satisfies the eigenvalue equation.

Table 1 Comparison of measured and computed natural frequencies

Mode No.	Natural frequencies, Hz Measured	Computed
1	14.18	13.83
2	21.50	22.78
3	23.26	23.83
4	24.36	24.37
5	24.63	26.10
6	26.29	28.76
7	27.66	28.90
8	28.67	29.77
9	29.22	29.87
10	33.42	29.96
11	34.49	30.79
12	35.37	31.85
13	41.04	35.58
14	42.11	38.46
15	42.99	
16	43.48	

The improved model (M,K) which deviates in a minimum fashion from the analytical model (M_A,K_A), contains the modes and frequencies Φ and Ω^2 which include the measured modes and frequencies ϕ_{I_i} and ω_i.

Application of Analytical Model Improvement

Description of Long Duration Exposure Facility Data

The AMI method was applied to the long duration exposure facility (LDEF), which will be a future Space Shuttle payload containing numerous trays of experiments. The configuration tested and analyzed was without trays. The structure is shown schematically in Fig. 1. A description is found in Ref. 21.

A NASTRAN model of the structure was developed, having 1097 degrees of freedom. This model subsequently was reduced to 508 degrees of freedom by eliminating the rotational degrees of freedom and certain axial degrees of freedom. It was the 508 degree of freedom model analytical M_A,K_A matrices that were used in these analyses. The ground vibration tests that were carried out identified 19 modes, each consisting of a set of displacements at 101 locations on the structure.

As an illustration of the reasonableness of the analytical model, Table 1 shows a comparison of measured natural frequencies and those obtained from analysis using the 1097 degree of freedom model. Additional natural frequencies and modes were computed; however, those shown are those which were identified as corresponding to the observed modes, i.e., the "major" modes. Note that the computed modes are not used in the analysis but are shown here for comparative purposes only. The frequencies are shown in numerical order

Table 2 Matrix changes without frequency terms in full mode computation

Modes[a]	Description	Mass matrix changes, %[b] δ_1	δ_2	δ_3	Stiffness matrix changes, %[b] δ_1	δ_2	δ_3
7,8,10,15	All vertical	0.94	0.04		0.11	0.07	
1,3,4,9	All torsion	1.73	0.13		0.08	0.05	
2,5,11,12	All lateral	0.45	0.02		0.09	0.05	
All above		6.83	0.46		0.16	0.15	
6,13,14,16	Misc. coupled	2.10	0.11	0.13	0.10	0.07	0.08
1-4	By frequency	1.64	0.11	0.11	0.08	0.04	0.08
5-8		2.02	0.08	0.10	0.11	0.06	0.11
9-12		0.40	0.02	0.03	0.08	0.05	0.05
13-16		0.99	0.06	0.06	0.11	0.08	0.08
1-16		14.74	1.24	0.95	0.20	0.24	0.18

[a]See Table 1. [b]See Eqs. (15-17).

Table 3 Matrix changes with frequency terms in full mode computation

Modes[a]	Description	Mass matrix changes, %[b] δ_1	δ_2	δ_3	Stiffness matrix changes, %[b] δ_1	δ_2	δ_3
1-4	Low frequencies	1.20	0.03	0.06	0.06	0.03	0.07
5-8	Higher frequencies	57.8	2.23	2.23	0.07	0.10	0.12
9-12	Higher frequencies	0.64	0.04	0.04	0.07	0.04	0.06
13-16	Highest frequencies	1.63	0.06	0.10	0.06	0.02	0.06
5,7-9	6 omitted	56.3	1.9	1.9	0.09	0.08	0.10
5,6,7,9	8 omitted	4.2	0.23	0.24	0.06	0.03	0.08

[a]See Table 1. [b]See Eqs. (15-17).

Table 4 Matrix changes with frequency terms in full mode computation using 19 modes

Modes[a]	Description	Mass matrix changes, %[b] δ_1	δ_2	δ_3	Stiffness matrix changes, %[b] δ_1	δ_2	δ_3
15-18		17.31	1.07	1.05	0.08	0.11	0.12
15,16,17,19	18 omitted	1.19	0.05	0.07	0.07	0.03	0.11
15,16,18,19	17 omitted	1.59	0.06	0.08	0.07	0.03	0.11
1-19	All modes	74.01	4.04	2.91	0.16	0.26	0.23
1-19	8,18 omitted	5.84	0.54	0.37	0.13	0.21	0.20

[a]See Table 1. [b]See Eqs. (15-17).

and no correspondence between the measured and computed modes is either implied or necessary.

Description of Analyses

The application of Eq. (4) to obtain the full modes required the inversion of a matrix of order 407 (i.e., 508-101). This was performed once when the frequency dependent terms were ignored and once for each mode when these terms were included. A decomposition algorithm[17] was used to increase the efficiency of this computation.

In order to become familiar with the sensitivity of the process to the number and types of modes, a number of analyses were carried out using from 4 to 19 modes. In Eq. (7), the order of m_A ranged from 4 to 19. Other operations in Eqs. (7) and (14) involved matrix multiplications of order $(508 \times 508) \times (508 \times 19)$, in the worst case.

Measures of Changes

The evaluation of the changes in the matrices is necessary if one is to pass judgement on the acceptability of the improved model. No single measure of these changes can be completely meaningful. Therefore, three separate parameters were computed to assist in this evaluation. The first, δ_1, is the root mean square (rms) of the element changes divided by the rms of the elements of the original matrix. Thus, if C represents the matrix of changes and A represents the analytical matrix,

$$\delta_1 = \left(\sum_i \sum_j C_{ij}^2 \Big/ \sum_i \sum_j A_{ij}^2\right)^{1/2} \tag{15}$$

This measure makes no recognition of the relative changes of the elements. In a sense, the diagonal elements may be considered to be indicators of the magnitude of the data in the corresponding rows and columns. Thus, two other measures of changes were computed. One is simply the rms of the relative changes in the diagonal elements

$$\delta_2 = \left(\sum_i C_{ii}^2 \Big/ A_{ii}^2\right)^{1/2} \tag{16}$$

The third measure is somewhat more general and relates all the changes to corresponding diagonal elements

$$\delta_3 = \left[\sum_i \sum_j C_{ij}^2 \Big/ A_{ii} A_{jj}\right]^{1/2} \tag{17}$$

In the following data, it will be seen that for the mass matrix, δ_1 is generally very much larger than δ_3. For the stiffness matrix, δ_1 and δ_3 are quite close in magnitude. δ_2 and δ_3 are nearly equal in both cases. The difference in δ_1 and δ_3 for the strongly diagonal mass matrix is indicative of a large number of off diagonal changes which are large compared to the original elements but small compared to the corresponding diagonal elements. It is believed that δ_3 is more meaningful of the measures of change.

Summary of Analysis Results

The mass and stiffness matrices were improved using several sets of the measured modes and with and without the frequency dependent terms in the full mode computations in order to gain some insight into the characteristics of the analysis.

A group of analyses were performed using the first 16 modes without frequency dependent terms and the results are shown in Table 2. This analysis is more efficient since only one inverse of the large matrix in Eq. (4) need be performed. The measured modes were divided into those which appeared to be primarily vertical or lateral bending or torsion as indicated in Table 2. Also a group of modes that contained significant displacements in more than one direction are labeled as "misc. coupled." Each of these sets of modes was used in the analysis along with a grouping by frequency and then all 16 of the modes were included. In each case, the resulting M,K represents a model having as solutions the particular set of modes and frequencies that were input.

The data in Table 2 appears to contain no anomalies and the greater the number of modes used, the greater the changes that are required in the matrices. Note that in the worst case, using all 16 modes, the most meaningful measure, δ_3, indicates a change of less than 1.0% in M_A and less than 0.2% in K_A.

Some of the conditions analyzed were repeated using the more exact frequency dependent analysis. These data are shown in Table 3. It is noted that modes 1-4, 9-12, and 13-16 give changes which are of the same order as those of Table 2. However, modes 5-8 result in large changes in the mass matrix. In order to determine if this was caused by a single mode, the analysis was rerun, first omitting mode 6 and then mode 8. In each case mode 5 was added to keep the number of modes at 4. Omitting mode 6 had little effect, but omitting mode 8 reduced the changes significantly.

The analyses of Table 3 were continued using all 19 measured modes. The data is presented in Table 4. A similar anomaly to that involving mode 8 was observed when modes

Table 5 Generalized mass matrix of full measured modes[a] and analytical mass matrix[b]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1.00																		
2	0.07	1.00																	
3	0.06	−0.12	1.00																
4	−0.06	−0.04	−0.01	1.00															
5	−0.07	−0.06	−0.05	−0.25	1.00														
6	0.08	−0.14	−0.01	0.10	0.18	1.00													
7	0.14	−0.08	0.02	0.02	0.15	0.48	1.00					(Symmetric)							
8	0.07	−0.02	0.00	−0.04	0.13	0.33	0.90	1.00											
9	0.01	−0.12	0.04	0.10	0.13	−0.03	−0.23	−0.33	1.00										
10	−0.05	0.04	−0.04	−0.03	−0.05	−0.16	−0.09	0.00	−0.06	1.00									
11	0.02	−0.01	−0.06	0.00	0.03	0.13	0.07	−0.02	0.05	−0.04	1.00								
12	0.06	−0.05	−0.01	0.00	0.03	0.08	−0.01	−0.05	0.06	−0.06	0.11	1.00							
13	−0.16	0.05	−0.03	−0.01	−0.02	−0.21	−0.03	−0.02	−0.01	0.10	−0.03	−0.01	1.00						
14	0.00	0.02	−0.01	0.00	−0.02	−0.06	−0.02	0.00	−0.01	0.03	−0.05	−0.04	0.17	1.00					
15	0.09	0.00	0.01	0.00	−0.02	0.04	−0.03	−0.01	0.02	0.01	−0.03	0.05	−0.16	0.03	1.00				
16	−0.03	−0.01	−0.01	0.00	0.00	−0.06	0.00	0.01	0.02	0.01	−0.02	0.09	0.12	0.01	0.14	1.00			
17	−0.17	−0.03	0.01	0.00	−0.04	−0.01	0.00	−0.02	−0.02	−0.03	−0.09	−0.07	0.03	0.11	−0.11	0.00	1.00		
18	−0.14	−0.03	0.03	0.00	−0.03	0.04	0.01	−0.03	−0.03	−0.06	−0.09	−0.10	0.09	0.09	−0.09	−0.01	0.84	1.00	
19	0.07	−0.03	−0.08	0.01	−0.03	0.02	0.01	0.02	0.05	−0.03	−0.02	0.13	−0.03	−0.01	0.10	−0.01	−0.11	−0.21	1.00

[a] Computed with frequency dependent terms. [b] Underlined elements indicate strong coupling.

17 and 18 were included. Eliminating either of these modes significantly reduced the magnitude of the changes. When modes 1-19 were included the mass matrix changes were relatively large. If one concluded that the measured modes 8 and 18 were not valid and omitted them from the analysis, the changes in M_A and K_A are $\delta_3 = 0.37\%$ and $\delta_3 = 0.20\%$, respectively.

Further insight into the validity of the modes may be gained from examination of Table 5, the full generalized mass matrix m_A, using the analytical mass matrix M_A. It is apparent that strong coupling exists between modes 7 and 8 and between 17 and 18, and coupling to a lesser extent between 6 and 7 and 6 and 8.

Concluding Remarks

A procedure has been developed which finds minimum changes in an analytical model to make it exactly agree with a set of measured modes and frequencies. The procedure does not require iteration or eigenanalysis. It is suitable for application to large models and where there are fewer test points than degrees of freedom in the model.

There are indications that measured "modes" which are not truly independent may be identified. This characteristic of the method is inferred from limited data. It also appears that it is important to use the frequency dependent terms in Eq. (4) rather than the simpler form which is equivalent to commonly used model reduction procedures. It is necessary that engineering judgement be applied in evaluating the reasonableness of the minimum changes and the exclusion of measured modes. Nonacceptance of the improved model implies significant errors in the model or the test data.

The improved model is expected to be a better base for other dynamic analyses, such as: coupling to other components, studies of effects of structural changes, changes in boundary conditions, and response to applied loads.

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